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ON ULTRASONIC WATTAGE AND SPECTRAL NUMBER RELATIONSHIP

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ABSTRACT. An experimental study has been made between the input wattage and the maximum number of spectral orders for different coils and different circuits. The nature of variation of the order number against the wattage is described by an empirical relation of the form $w = bn^2 + cn^3$. It has been observed that the relation obtained by Raman and Nagendra Nath, when suitably analysed, is in complete accord with the experimental results. A possible reorientation of Raman and Nagendra Nath's theoretical relation to suit this aspect of ultrasonic study has been suggested.

INTRODUCTION

The diffraction spectra of ultrasonic waves obtained by the method of Debye and Sears ((1932) have been studied from various aspects. Bär (1932) studied the spectra critically from various aspects and incidentally established that the sound intensity leads to the gradual appearance of the spectra of higher orders. Raman and Nagendra Nath (1936) gave a general theory of the intensity relationship of the spectra of different orders, which showed that higher orders of spectra are expected with greater sound intensity. A definite relation between the sound intensity and the observable spectral number was not easily derivable from the above theory and no definite experiment to formulate the relationship has been tried.

It is well known that it is difficult to have an exact measure of the ultrasonic energy in a liquid. It is, however, easily possible to have a measure of the relative ultrasonic energies from either the input or the output energies of the oscillating circuit, assuming that the conversion factor between input and output and also between output and ultrasonic energy remains unchanged. That such a constancy of the conversion factor holds good, will be apparent from the discussion afterwards. It was also checked by observing a linear relation between the input wattage and the output power measured by an absorption wave meter, placed at a definite position relative to the oscillatory circuit.

According to expectation it has been observed by us that the number of spectral orders increases gradually with the input wattage. The experimental relation between the wattage and the observed spectral number has been found,

in the following, to be of the form $w = bn^2 + cn^3$ where 'w' is the input electrical wattage, 'n' the number of observed spectra and 'b' and 'c' are constants of the circuit and the system. It has been further possible to examine the relationship between wattage and spectral number from the standpoint of Raman's theory and it has been shown that there is exact correspondence between the expectation from this theory and the experimental results, throughout the range of variation of wattage, at different frequencies and with different oscillatory circuits. Incidentally this shows the possibility of reorientation of Raman and Nath's relationship to connect ultrasonic power or refractive index variation with the observed spectral number.

EXPERIMENTAL ARRANGEMENT

The experimental arrangement comprises of the method of production of intense ultrasonic beams at different frequencies, the arrangement of variation of input electrical wattage and the method of observing diffraction pattern by ultrasonics.

For the production of ultrasonic beams at different frequencies a piezo-electric quartz of natural frequency of 1 mega c/s was vibrated to its higher odd harmonics by the aid of electric oscillations of an electric generator.

The electric generator consisted of a full wave rectifier and an oscillating circuit of the Hartley type. The oscillating circuit was made up of an adjustable self induction and a variable condenser. It was tuned to the natural frequency of the quartz and to its higher odd overtones by varying the capacity and the tappings in the coil. Whenever a particular overtone was not obtainable by tapping from the scope of the winding of the coil, the coil was replaced by a new one. To obtain lower harmonics with a particular coil it was also sometimes necessary to extend the range of the capacity by joining a new capacity in parallel with it.

We used a 100-watt generator. When used with full power the input was 110 v. AC. To vary the power of vibration of the quartz, the input voltage to the power transformer only was varied, thus changing the input wattage with the help of a variable rheostat. An accurate voltmeter in parallel and an accurate ammeter in series were connected to measure the input wattage accurately. The filament of the oscillator was heated by a separate transformer.

The vibrating quartz was put inside a rectangular glass vessel, filled up with pure transparent kerosine. The vessel was suitably covered with a glass plate to minimise evaporation. It was placed on a prism table of a spectrometer. The ultrasonic beam inside the liquid was illuminated with the properly collimated light of a monochromatic sodium lamp. The relative positions of the collimator, the glass vessel and the quartz were undisturbed during the experiment to ensure constant condition of visibility of the spectra. The quartz was

placed in such a way that the diffraction spectra became symmetrical with reference to the undeviated ray. The spectra were observed by the telescope of the spectrometer from the opposite face of the vessel.

EXPERIMENTAL RESULTS

The circuit was tuned to the condition for the appearance of maximum number of spectra for each observation with different wattages and with different frequencies.

It was, however, noticed that the coupling of the differently wound coils with the condenser, affected considerably the appearance of spectra, even for the same frequency and wattage. To emphasise on this difference we took two sets of observations with two different coils. One was a thin steel coil, wound closely on silica base. The other was thick copper coil in air, wound with wide spacings. The former coil gave greater number of spectra i.e., sound intensity, than the latter for the same input wattage and the frequency. A clear comparison can be made between them from the tables. So the coupling condition is observed to be an influencing factor in spectra-input wattage relationship. With the thin coil which showed a larger number of spectra, we simply counted the number of spectra for different steps of wattages. It was difficult in such a case to adjust the wattage so that spectra would appear one by one. With the other type of the coupling, however, the number of spectra was considerably less. So the wattage was adjusted until a particular order of spectra just appeared. If for a given wattage the intensity of a particular order is visually greater than that at the time of appearance, it was marked with + and + + according to the condition of the intensity of the order. The readings for two different oscillating circuits are tabulated below :

TABLE I

Wattage-spectra observations with thin steel coil.

Frequency f = 3 Mc/s.		Frequency f 5 Mc/s.	
No. of fringes (n)	Wattage (w)	No. of fringes (n)	Wattage (w)
9	7.2	7	6.7
12	12	8	9.7
16	18.7	9	14.8
19	29.4	10	18.7
22	40.2	11	23.6
23	52	13	35.6
		15	58.9

TABLE II

Wattage-spectra observations with thick copper coil.

f=5 Mc/s.		f 7 Mc/s.		f 9 Mc/s.		f 11 Mc/s		f=13 Mc/s.	
n	w	n	w	n	w	n	w	n	w
2[1]	6.25	2[1]	7	2	8	1[1]	6 4	1	11
3	8	3	12.6	3	30	2	56	.	.
4	16	4	27	4	69	.		.	
5	28	5	58						
6	47	6	76	..	.				
7	72	..	..						

Plotting the number of spectra against wattage for a particular frequency, a smooth curve was obtained. For different frequencies, the curves retained the same nature of variation. From the nature of the curves it appeared that the spectra-wattage relationship is more or less of parabolic nature. On plotting n^2 against wattage, we observed that although with smaller values of n^2 the relation is linear, with higher values of n^2 the curves deviated from its straight path towards wattage axis. This deviation went on increasing with higher frequencies. This suggested that an n^3 term should also be taken in. It is also evident from the increasing deviation with the frequency that the coefficient of n^3 will go on increasing with the frequency. Thus, the suggested empirical relation is $w = bn^2 + cn^3$. This form of the relation is fitting the curves throughout at different frequencies within the experimental error. Below, in figure 1

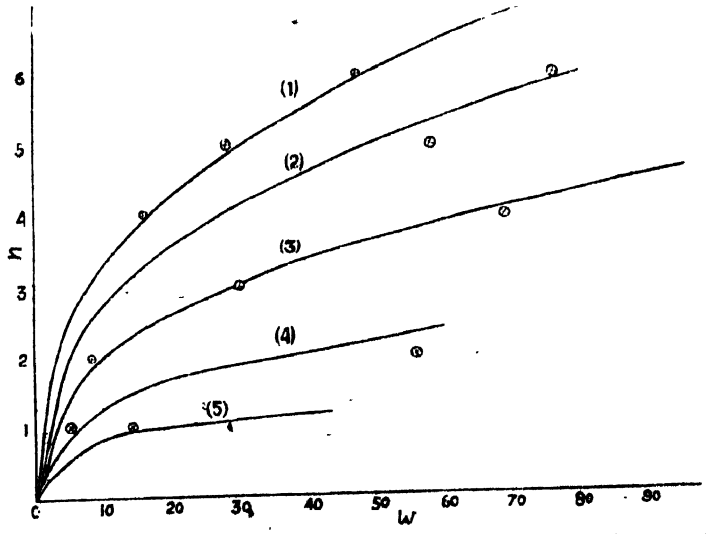

Fig. 1. Input wattage and fringe number graph. Graphs (1), (2), (3), (4), (5) correspond to the frequencies 5, 7, 9, 11, 13 Mc/s. respectively.

and figure 2 we have drawn all the theoretical curves with the above relation and experimental observations are indicated therein by points within small closed circles.

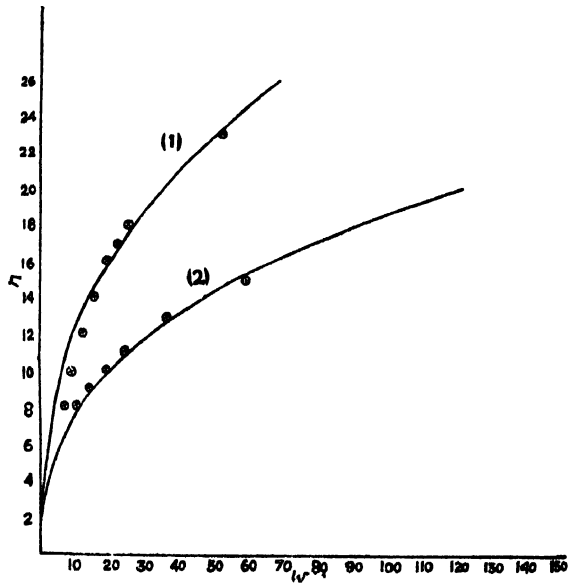

Fig. 2. Input wattage and fringe number graph. Graphs (1), (2), correspond to the frequncies 3Mc/s, 5Mc/s obtained with a different oscillatory unit.

The values of b and c went on increasing with the frequency. They must, naturally, be some function of frequency or of the order of the harmonics and also of the factors involving the constants of the circuit. A range of values for b and c were found to be possible to conform to the experimental curve at a definite frequency, within the range of experimental error. The variation of c with the frequency was very rapid and it was observed that a smooth curve between log c and the frequency limited very much the possibility of deviation of c values. A mean curve between log c and the frequency is drawn in the figure 3. This curve looked to be of logarithmic nature and loglog 100c being plotted in the same figure 3, with a changed scaleof Y-axis gave a linear curve. Thus the relation obtained between c and frequency was given by log $c=d+ke^{mf}$ where f is the frequency and k, m, d are constants depending on the nature of the circuit and the system.

Knowing the values of c for different frequencies, the corresponding values of b were found out. They came within the range of value of b previously determined. These determined values of b gave a smooth relationship with the frequency as shown in figure 4. The nature of the curve indicated a logarithmic relation. On plotting log b against frequency a straight line was obtained as shown in the same figure 4.

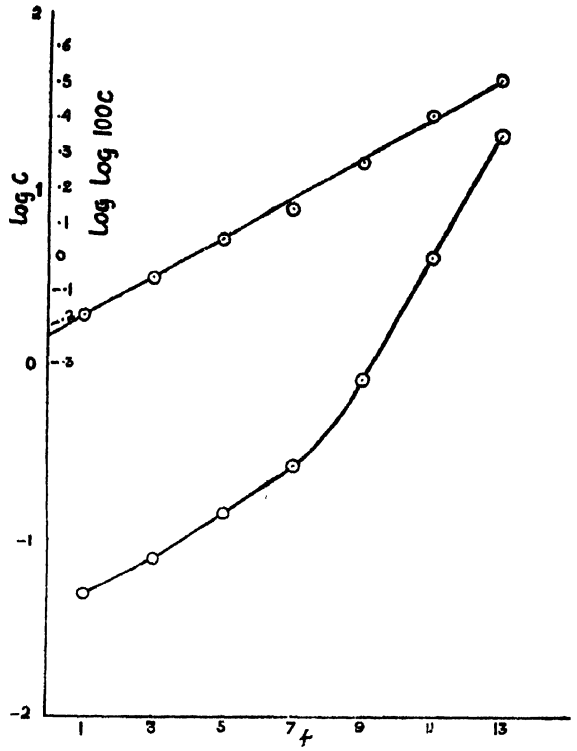

Fig. 3. Variation of the constant c with frequency shown in log $c-f$ and log $100c-f$ forms.

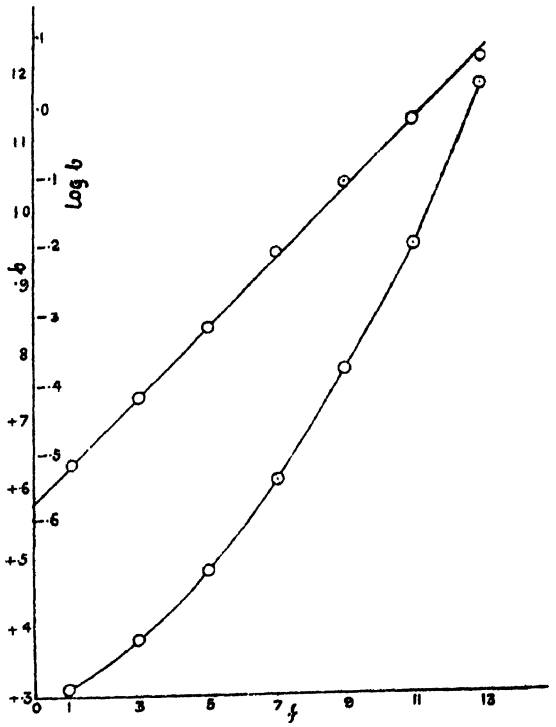

Fig. 4. Variation of the constant b with frequency shown in $b-f$ and log $b-f'$ forms.

Thus we have $b = A\,e^{Bf}$ where A and B are constants depending on other factors like k, and m. The determined values of b and c for different coils and frequencies are tabulated below :

TABLE III

	Thick copper coil		Thin steel coil.	
f in Mc/s	b	c	b	c
1	0 31	0.049	..	..
3	0 38	0.078	0.0407	0.00234
5	0.48	0.144	0.1	0.01
7	0.61	0.269	..	.
9	0 77	0.832	..	.
11	0 95	4.2	..	..
13	1 18	20 8	.	.

DISCUSSION OF RESULTS

It has been already stated that Raman and Nagendra Nath's theory is apparently not suitable to give us a relationship between the spectral number and the ultrasonic wattage or the change of refractive index caused by it. This is particularly because the theory gives us the intensity of the order number 'n' by the Bessel function $J_n^2(v)$ where v contains the refractive index variation due to ultrasonics. Evidently from the form of the Bessel function, we shall obtain for a particular value of n, various values of v to give the same value of intensity $J_n^2(v)$. Also for a particular value of v, we would have various values of n to give the same value of intensity. This would be apparent from a careful study of the graphical representation of intensities of different orders for various values of v, as given by Raman and Nath. In such a case of multiple-valued relationship on both sides, it is not easily possible to find a suitable relation between the total observed order number and the values of v. If, however, from the multiple-valued v values we select the minimum value to give us the just observable intensity $J_n^2(v)$ for the n-th order, we would have a single-valued n corresponding to a single-valued v and naturally there would exist a suitable relation between them. This does not seem to be an easy task. Attempts are, however, being made here in this direction. To circumvent this difficulty, we have studied the calculated intensity picture for different values of v given by Raman and Nath and also by Levi (1936), on the basis of Raman and Nath's theory, and have obtained the minimum value of v for the appearance of a given order number. On plotting these order numbers (appearing for

the first time with increasing v values) against the v values, we obtain a curve between the maximum number of fringes and the values of v or the refractive index change on the basis of Raman and Nath's theory. The single-valued relationship between the n-th fringe with the just observable intensity and the minimum value of v suggested before would be the equation for the above mentioned curve. This graph has been plotted in Figs. 5 and 6, with the help of the intensity diagrams of different orders for various values of v as given in the work of Raman and Nath and also of Levi. Since the ultrasonic wattage is taken as proportional to the input wattage and also since ultrasonic wattage may be justifiably taken to be proportional to the square of the pressure change or to the square of the change of refractive index of the medium as has been established by the work of Sanders (1936), we may consider that the v values of Raman's work may be replaced by $K\sqrt{w}$ where K is a constant for a set up.

In accordance with Raman and Nath's theory, then, we should expect that the graph between the observed number of fringes and $K\sqrt{w}$, with a suitable choice of K, should be identical with the graph mentioned above between v values and the maximum number of fringes. The identity of our experimental curves throughout the range of wattages for different frequencies and different coils with the curve obtained on the basis of Raman and Nath's theory, will be clear from a perusal of Figs. 5 and 6.

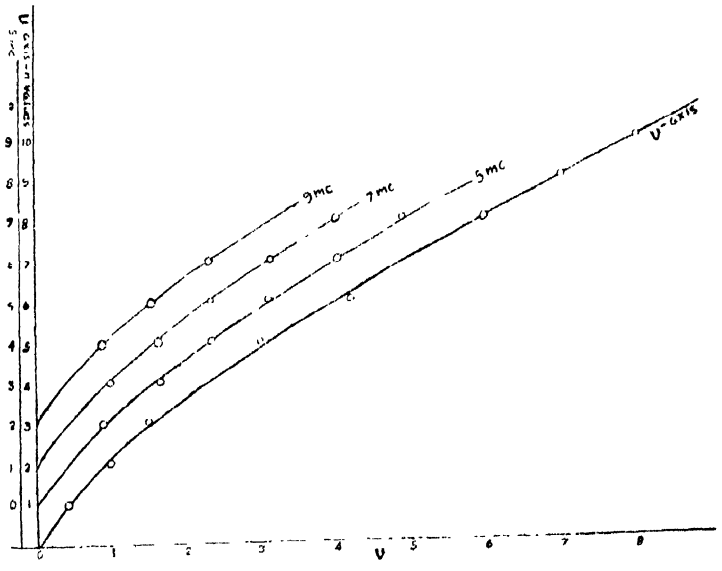

Fig. 5. The points in the lowest graph are taken from Raman and Nagendra Nath's diagrammatic representation of intensities and the smooth curve through them have been shifted to different scales of representations, where the points indicate the observed fringe numbers against $K\sqrt{w}$ for different frequencies. The k values for 5,7,9 Mc/s. are 0.6, 0.45, 0.28 respectively.

In figure 5, the lowest curve is from Raman and Nagendra Nath's diagrammatic representation of intensity for various v values, the points being taken from the records of the diagrams. The upper graphs have been drawn by shifting the y-axis through unity in steps, to avoid confusion, as would be clear from

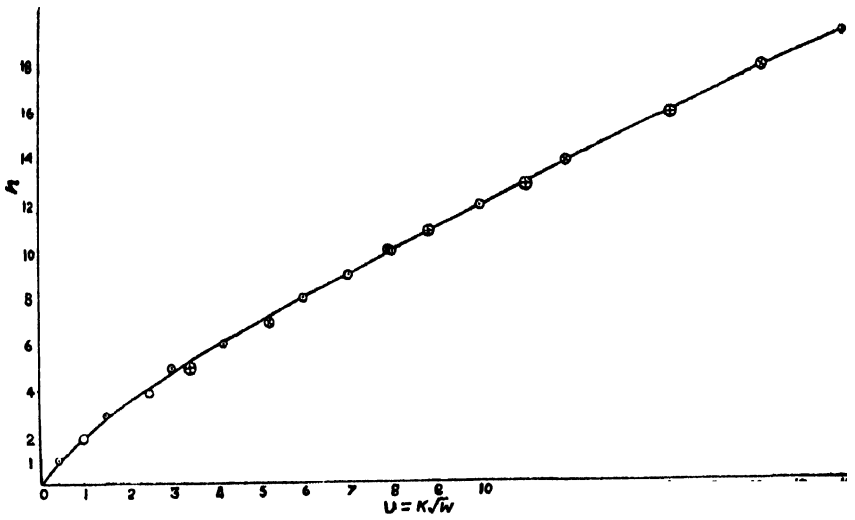

Fig. 6. The circles with points are taken from Raman and Nagendra Nath's and also from Levi's diagramatic representations of intensities. The circles indicated as × and + refer to the observed values of fringe number against $K\sqrt{w}$ for the frequencies 3 and 5 Mc/s with a different coil. The k values are 3.24, 1.75 respectively.

the scales alongside. The continuous lines are again the curves from Raman and Nath's tabulated values and the points are plots of the observed numbers against $K\sqrt{\omega}$ with adjusted constant K for different frequencies They have been previously represented in Fig. 1 in a different way Similarly in Fig. 6 the full curve is obtained from the minimum v values and the observed numbers from Raman and Nath's and also from Levi's diagrammatic intensity relations, where the points indicated as × and + are the plots of the observed numbers against $K\sqrt{w}$ for the 3 Mc/s and 5 Mc/s data respectively. These have been represented before in graph 2. As will be apparent from the graphs, the agreement is remarkably good, which signifies that Raman and Nath's theory is substantiated completely on this aspect and in this range of experimental study. It also establishes a linear relationship between input wattage, the output wattage and the ultrasonic wattage, as otherwise the agreement would not have been obtained.

The only drawback of Raman's theory to deal with this aspect of the experiments is the non-availability of a suitable relation connecting the maximum

number of fringes and the values of v or of the variation of refractive index. It is also apparent from our empirical relationship that the relation between the n values, obtaining a particular small fixed value of $J_n^2(v)$, and the minimum v value should be reducible to the form $v = \sqrt{Cn^2 + Dn^3}$. Otherwise an alternative relation should be found to describe the minimum v against observed spectral number graph as obtained from Raman's work. This would automatically transform to the wattage against observed spectral number relationship as described by the graphs 1 and 2.

It may be further pointed out that in the $K\sqrt{w}$ values for different frequencies and coils which identify with the v values of Raman and Nath's theory, the constants K lie in the range from 10 to 10^{-1}. As vi s given by the expression $\frac{2\pi\Delta\mu l}{\lambda}$ where l, the depth of light path through ultrasonic beam is about 2 cm in our experimental condition and λ the wave length of light is about 6×10^{-5} cm, we note that the conversion factor from the square root of input wattage to the refractive index variation is of the order of 10^{-5} to 10^{-6}. This is not an unexpected order of conversion factor.

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Letter to the Editor

RAMAN SPECTRA OF SOLUTIONS OF O-DICHLORO-BENZENE IN METHYLCYCLOHEXANE

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It was observed recently (Biswas, 1955) in an investigation on the Raman spectra of dichlorobenzenes in different states and at different temperatures that two intense lines appear at 469 cm^{-1} and 487 cm^{-1} in the Raman spectrum of *o*-dichlorobenzene in the liquid phase and when the liquid is solidified the two lines are replaced by an intense single line at 478 cm^{-1}. It was suggested that the line 469 cm^{-1} might be due to dimers present in the liquid and the corresponding line due to the single molecule was at 487 cm^{-1}. As solvent molecules have much influence on molecular association in such a case it might be expected that if the substance would be dissolved in suitable solvents the relative intensities of the two lines due to the monomer and the dimer respectively would change appreciably. Recently, methylcyclohexane has been found to be a very good solvent for this purpose (Sirkar and Kastha, 1955) and therefore, the Raman spectra of solutions of *o*-$C_6H_4Cl_2$ in methylcyclohexane have been investigated to test the above hypothesis.

A comparison of spectrograms due to the pure liquid and the solutions of this substance in methylcyclohexane reproduced in figure 1 shows that the

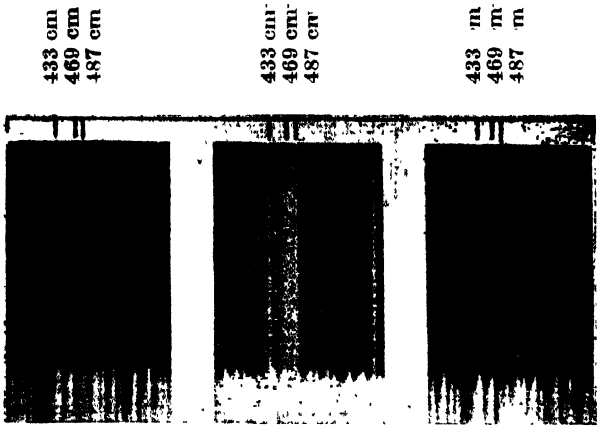

Fig. 1

solvent has great influence on the relative intensities of the two lines mentioned above. The integrated intensities of the two lines at 469 cm^{-1} and 487 cm^{-1} are

nearly the same in the Raman spectrum due to the pure liquid at room temperature. The spectrogram due to a 40% solution of this substance in methylcyclohexane shows that the intensity of the line 469 cm^{-1} is less than half that of the line at 487 cm^{-1}. When the strength is reduced to 20% by volume, the line 469 cm^{-1} becomes broader and appears as a very weak and diffuse satellite of the intense line at 487 cm^{-1}. Again, on comparing the intensities of both these lines with the line at 433 cm^{-1} it can be seen clearly that the line 469 cm^{-1} becomes much weaker and the line 487 cm^{-1} becomes relatively much stronger when *o*-$C_6H_4Cl_2$ is dissolved in methylcyclohexane. It is thus evident that in dilute solutions of methylcyclohexane the Raman line at 487 cm^{-1} becomes stronger at the expense of the other line at 469 cm^{-1}.

The above results furnish conclusive evidence in support of the suggestion (Biswas, 1955) that the line 469 cm^{-1} of the liquid *o*-$C_6H_4Cl_2$ is due to a dimer and the line 487 cm^{-1} is due to the same mode in the single molecule. The line 487 cm^{-1} was attributed to the vibration of the benzene ring of mode ϵ_g^+ (Nordheim and Sponer, 1943). In the liquid state this vibration is influenced by the field of the neighbouring polar molecules of the substance and a second line corresponding to the same mode in a dimer formed in the liquid state is produced at 469 cm^{-1}. The relative intensities of these two lines suggest that in the liquid state at room temperature nearly 50% of the molecules are in the associated state. When the strength of this solution is only 15% by volume, the number of associated molecules becomes negligibly small and the intensity of the line at 469 cm^{-1} due to the dimer is reduced to a very low value.

One of the three lines in the neighbourhood of 200 cm^{-1} observed in the spectra due to *m*-dichlorobenzene, *m*-chlorotoluene and *m*-bromotoluene was attributed to dimers (Biswas, 1955). This line, however is found to persist even in the case of very dilute solutions of these substances in methylcyclohexane. An alternative assignment is, therefore, to be found as the line seems to be due to the single molecule.

In conclusion the author wishes to acknowledge his indebtedness to Prof. S. C. Sirkar, D.Sc., F.N.I., for his kind help and guidance during the progress of this work.

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